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Analysing the Tasks in Middle School Mathematics Textbooks According to the Levels of Cognitive Demand

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Abstract

This study aims to investigate and categorize the tasks in middle school mathematics textbooks based on their cognitive demand levels. The goal is to examine how these tasks align with educational objectives aimed at fostering students' critical thinking and problem-solving abilities. This research employed the document analysis method. This study evaluated the mathematical tasks in 5th to 8th grade textbooks used in schools in Türkiye, using Smith and Stein's (1998) framework for classifying mathematical tasks. The data were examined using the content analysis method. The results indicated that most tasks in these textbooks were concentrated at the levels of procedures without connections and procedures with connections. Additionally, the proportion of tasks requiring higher-order cognitive skills, specifically, doing mathematics tasks, was relatively low. Based on these findings, future textbook revisions should integrate more high cognitive demand tasks to both strengthen students' procedural skills and foster complex problem-solving and critical thinking abilities.

Keywords: Cognitive demand, mathematical task, textbook, middle school mathematics, activity.

Introduction

Despite various criticisms (Radford, 2008), the constructivist approach, gained influence in mathematics education since the mid-1980s, fundamentally emphasizes studentcenteredness and active student participation in the learning process. In this approach, students are expected to restructure new knowledge by building on their existing knowledge, essentially constructing new information on the foundation of what they already know (Toluk-Uçar, 2020; Van de Walle et al., 2023). Boston et al. (2017), argued that the key element in achieving the goals of the constructivist approach and supporting students' meaningful understanding of mathematics is the incorporation of activities that promote problem-solving and logical reasoning in the teaching process. Activities can be defined as processes that encourage active student participation or they can be defined as individual or series of tasks selected by the teacher but voluntarily performed by the student (Dede et al., 2020). Similarly, Mathematics activities shape students' learning and experiences in mathematics classes (Johnson et al., 2017). Well-organized and effectively implemented mathematical activities allow students to reflect on their knowledge, skills, and understanding within the classroom (Smith & Stein, 2011). Kilpatrick et al. (2001), state that clear and motivating mathematical activities are one of the three conditions necessary for students to demonstrate their logical reasoning abilities (the other two are having sufficient knowledge and being familiar with the content). When examining the term "activity" within the mathematics education literature, it appears in various forms.

Although some do not fully capture the intended meaning, the terms "exercise", "task", and "activity" are often used interchangeably to refer to activities. The typical characteristics of exercises are as follows: teachers are highly active and they want to control process, teachers directs the students with minimal student participation, the content and goals of the lesson are clearly known in advance, they were proceed in a written format, the focus is on procedural skills rather than conceptual learning, and the aim is to reinforce learned or memorized content through repetition (Cichy et al., 2020; Foster, 2018; Watson & Mason, 2006). Tasks,

on the other hand, have different characteristics. They involve low teacher control, but students are guided under teacher supervision, and neither the teacher nor the student is central. Active participation is not mandatory, and tasks can be presented in any format. They focus on both procedural and conceptual knowledge, involving repetition of learned content through social processes such as peer support and collaborative learning. Although the topics and goals are clear, they are not explicitly stated. (Chapman, 2013; Foster, 2013). Lastly, activities are characterized by student control with the teacher acting as a guide, where the student is central, and active participation is required. The goals and content to be learned are not explicitly given at the beginning, and activities are presented in a physical format. They prioritize building procedural skills on conceptual learning and involve constructing new topics based on previous learning. Additionally, they encourage exploration (Antonijević, 2016; Ponte et al., 2014).

Considering the use and function of these three terms as a whole, activities can be defined as learning engagements that allow students to focus on specific mathematical ideas under particular pedagogical approaches and mathematical procedure. In this context, the fundamental basis of the term "task" can be said to be the provision of opportunities and conditions that help develop students' mathematical thinking, reasoning, modelling, and understanding skills in mathematical learning environments and teaching processes (Seah & Horne, 2021; Stein et al., 1996; Wess et al., 2021). Activities can be quite broad in scope; they may include; outdoor learning activities, projects or games and also questions from textbooks and worked-out examples, all of which fall within the concept of an activity.

Given the significant role of textbooks in shaping classroom activities, it is crucial to examine the cognitive demand levels of the tasks they contain to better understand how they align with educational objectives. This study aims to analyse the tasks in middle school mathematics textbooks and classify them according to their cognitive demand levels. By doing so, it seeks to provide insights into the nature of tasks presented in textbooks and their alignment with the educational goals of fostering critical thinking and problem-solving skills among students. The findings from this study will contribute to understanding the extent to which textbooks support or hinder the development of these essential skills in mathematics education.

Background and Related Context

Classifying tasks according to their cognitive levels or the cognitive demands expected from students is one of the useful methods to ensure that instructional materials target different levels of thinking, from basic recall to higher-order problem-solving, helping educators design more effective learning experiences. The classification of academic tasks according to their levels of cognitive demand, as the cognitive processes required for a student to complete a given task, was introduced by Doyle (1980, 1983). In his studies, Doyle (1983) categorized academic tasks into four groups based on their cognitive demand levels. These are: "memory tasks, procedural or routine tasks, comprehension or understanding tasks, and opinion tasks" (pp. 7-8).

The most typical feature of memory tasks is that students recognize or reuse previously seen or learned information in similar, identical situations. Memorizing the multiplication table is a good example of memory tasks. Procedural or routine tasks aim to have students

reach a result by using a specific or predictable formula. Solving addition or subtraction problems using a learned algorithm can be examples of such tasks (Dorner & Ableitinger, 2022). Comprehension or understanding tasks, on the other hand, require students to follow a three-step process. First, students need to recognize the reorganized forms of previously learned information. Then, they must apply the appropriate procedure from various options in a new situation. Finally, they are expected to make inferences for their possible future learning. Solving a routine real-life problem using several different strategies can be an example of these types of tasks (Plath & Leiss, 2018). In the final stage, opinion tasks require students to make choices among possible options with logical and sufficient justifications. A task that involves evaluating and selecting among various types of individual retirement plans based on variables such as monthly fees, duration, and repayment options can be presented as an example for opinion tasks (Leiss et al., 2019).

Figure 1.Characteristics of Mathematical Instructional Tasks

Low Cognitive Demand	Memorization	 Involves reproducing previously learned facts, rules, formulas or definitions or committing these to memory. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. Is not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. Has no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned.
Low Cognit	Procedures <u>without</u> Connections	 Is algorithmic. The use of a procedure either is specifically called for or is evident from prior instruction and/or experience. Requires limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. Is not connected to the concepts or meaning that underlie the procedure being used. Is focused on producing correct answers. Requires no explanation or explanations focus solely on describing the procedure that was used.
ive Demand	Procedures <i>with</i> connections	 Focuses students' attention on the use of procedures for the purpose of developing deeper understanding of mathematical concepts and ideas. Suggests explicit and/or implicit pathways to follow that involve the use of broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms. Can usually be represented in multiple ways, including the use of manipulative materials, diagrams, and symbols. Making connections among the representations helps students develop meaning. Requires some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students are engaged in conceptual ideas that underlie the procedure and develop understanding.
High Cognitive Demand	"Doing" Mathematics	 Requires complex, non-algorithmic thinking. Requires students to explore and understand the nature of mathematical concepts, processes, or relationships. Demands students do some type of self-monitoring or self-regulation of their own cognitive processes. Requires students to access relevant knowledge and experiences and make appropriate uses of them in working through the task. Requires students to analyse task constraints that may limit possible solution strategies or solutions. Requires considerable cognitive effort and may cause some level of anxiety for the students as they are working through the problem.

Similarly, mathematical tasks range along a continuum from routine exercises to complex and challenging problems. Based on Doyle's (1988) work, Stein and her colleagues conducted a series of studies to classify the types of mathematical tasks and the levels of

thinking required to solve them (Stein et al., 1996; Stein & Smith, 1998). Smith and Stein (1998) categorized mathematical tasks into two levels: lower-level and higher-level. The lower-level tasks were divided into memorization and procedures without connections, while the higher-level tasks were divided into procedures with connections and doing mathematics.

Figure 1 presents the characteristic features of each type of tasks based on their cognitive demand levels in detail (Smith & Stein, 1998). According to the classification presented in this figure, tasks that force students to engage in active inquiry and investigation, or that require building the processes and methods used to reach a result on a particular concept or idea, are considered tasks with high cognitive demands. On the other hand, tasks that encourage students to repeatedly use formulas, solutions, and algorithms without understanding the reasons behind them, or that require the use of memorized information or previously tried situations, are seen as tasks with low cognitive demands (National Council of Teachers of Mathematics [NCTM], 2014). The features specified in this classification can serve as a rubric for deciding whether to use pre-constructed or readily available tasks in the mathematics teaching process. They can also guide the development of original activities for teaching mathematics (Smith & Stein, 1998). Besides these functions, the different cognitive demand levels of tasks can be used to assess students' learning and mathematical understanding by examining their ability to complete these tasks (Van de Walle et al., 2023).

In their literature review on mathematics tasks, NCTM (2014) identified three main findings. The first is that not all tasks provide students with the same opportunities for thinking and learning. In other words, not every task used in the classroom has the expected impact on students. The second finding is that students' mathematics learning is maximized when tasks consistently promote higher-order thinking and logical reasoning, whereas it remains minimal when tasks consist of procedural repetitions. The third finding is that the most challenging type of tasks to implement in the classroom are those that require high cognitive demands, which are essential for achieving the highest levels of mathematics learning. Consequently, tasks in the mathematics teaching process are often transformed into types that require lower cognitive demand. One of the fundamental challenges in mathematics education is the inability to replace routine exercises, which can be solved by repeatedly applying a specific method and focusing solely on procedural skills, with tasks that emphasize conceptual learning. These routine exercises often fail to prioritize the development of deeper understanding in students (Lithner, 2017). In parallel with these findings, two main factors contribute to the reduction of cognitive expectations in tasks: students may request that teachers lower the difficulty level of tasks to achieve solutions through more straightforward, understandable steps, or when teachers notice that students are struggling with the tasks, they may take on some of the steps themselves by telling or showing students what to do (Kilpatrick et al., 2001).

The challenges in transitioning to higher-order cognitive tasks are also reflected in more recent studies. In the study conducted by Lee (2022), the cognitive demand levels and question types of tasks in 5th and 6th grade mathematics textbooks were analysed. The results showed that the proportion of tasks requiring mathematical reasoning was the highest, while the proportion of problem-solving tasks was the lowest. It was also found that the proportion of high-level tasks was quite low, and tasks requiring students to engage in deep cognitive

processes were insufficiently provided. Additionally, the analysis of the sub-questions of high-level tasks revealed that reasoning questions had the highest proportion. The study suggests that to develop students' mathematical thinking skills, a broader range of tasks requiring deep cognitive complexity should be implemented. Moreover, it emphasizes the importance of maintaining the cognitive level when creating sub-questions for high-level mathematical tasks.

Additionally, the study conducted by Ni et al. (2018), found that mathematical tasks involving high cognitive demand, multiple representations, and multiple solution methods enhanced students' abilities to solve complex problems. However, it was determined that these types of tasks did not have a direct impact on cognitive learning outcomes such as computation, routine and complex problem solving. Conversely, high cognitive demand for mathematical tasks positively influenced students' interest in mathematics, their classroom engagement, and dynamic views on learning mathematics. The study highlighted that the cognitive demand levels of mathematical tasks play a significant role in students' cognitive and affective learning outcomes, and that effectively implementing these tasks in the classroom encouraged students to form positive relationships with mathematics. When examining the literature from Turkish context on mathematics education, several studies have categorized the tasks or activities included in textbooks and curricula according to their cognitive demand levels. In their analysis of the algebra learning domain in the middle school mathematics curriculum, Ubuz et al. (2010) found that activities at the 6th, 7th, and 8th grade-levels included all types except for memorization, with approximately 60% of the activities requiring high cognitive demand across all three grade levels.

In a study focusing on textbooks, Ubuz and Sarpkaya (2014) discovered that the tasks in the algebra sub-learning domain of 6th grade textbooks were generally of the "procedures with connections" type. However, when examining the tasks used by mathematics teachers in the classroom, they found that most were of the "procedures without connections" type. Similarly, Engin and Sezer (2016) revealed that most tasks in the 7th grade mathematics curriculum and textbooks were at the level of "procedures with connections." Another study by Reçber and Sezer (2018) found that the proportion of tasks requiring high cognitive demand in the 8th grade textbooks was lower than expected in the middle school mathematics curriculum, with this discrepancy reaching up to 35% in some sub-learning domains.

Bozkurt and Yılmaz (2020) noted that the distribution of tasks in the 8th grade mathematics textbooks was almost equal between low and high cognitive demand levels. However, they found that these tasks were concentrated in the "procedures with connections" and "procedures without connections" categories. They also observed significant differences in cognitive demand levels between the two textbooks they examined. A more recent study conducted by Polat and Dede (2023) revealed that the tasks in the algebra learning domain of the analysed mathematics textbooks were generally of low cognitive demand. This indicates that the tasks do not require students to engage in high-level cognitive thinking and problem-solving skills when performing mathematical tasks.

When these previous studies are considered as a whole, it can be concluded that the tasks in the middle school mathematics curriculum are sufficient to provide students with a mathematical perspective and can promote logical reasoning and complex problem-solving

skills. On the other hand, it appears that the tasks in textbooks do not meet the expected levels of the curriculum and that there are differences between textbooks. In terms of classroom practices and the activities used, it is observed that tasks requiring low cognitive demand are predominantly preferred. Consequently, classroom implementations do not reflect the expectation of utilizing high cognitive demand tasks in the curriculum

Mathematics teaching processes heavily rely on textbooks as the primary resource for both teachers and students. Consequently, many of the tasks that teachers assign to students during mathematics lessons are derived from the tasks included in the textbooks. Despite various efforts to incorporate tasks that promote higher-order thinking and conceptual understanding, challenges persist in replacing routine exercises with more cognitively demanding tasks. Previous studies have highlighted that not all tasks or activities offer the same opportunities for student learning and thinking. While tasks that encourage higher-order thinking and logical reasoning are essential for maximizing students' mathematical understanding, such tasks are often underrepresented in classroom practices. Instead, there is a tendency to favour tasks requiring lower cognitive demand due to various constraints and challenges faced by teachers.

Method

Research Design and Source of Data

The aim of this study is to analyse and classify the tasks in middle school mathematics textbooks according to their cognitive demand levels. The design of the study is document analysis which examines written or printed materials to provide detailed information on the subject being studied (Bowen, 2009). In 2021, when the data for this study were collected, three 5th grade textbooks, three 6th grade textbooks, two 7th grade textbooks, and three 8th grade textbooks from different publishers were available in the Education Information Network [EBA] of the Ministry of National Education [MoNE]. However, the content, analysed in this study, includes tasks in the 5th grade (Ciritci et al., 2019), 6th grade (Çağlayan et al., 2019), 7th grade (Keskin-Oğan and Öztürk, 2019) and 8th grade (Böge and Akıllı, 2019) mathematics textbooks published by the MoNE. The study encompasses all tasks that students are assumed to solve during lessons, such as exercise questions, problems, solved examples, activities, and games. However, unit evaluation questions, which are typically assigned as homework, were excluded from the study. Therefore, a total of 1,802 tasks were analysed, including 372 tasks from the 5th grade textbooks, 477 tasks from the 7th grade textbooks, and 434 tasks from the 8th grade textbooks.

Data Analysis and Process

In this study, the mathematical tasks in the 5th, 6th, 7th, and 8th grade textbooks were evaluated based on the framework for classifying mathematical tasks, as explained in Figure 1, developed by Smith and Stein (1998). The data were examined using the content analysis method, as one of the techniques employed to identify concepts and relationships and derive conclusions from qualitative data (Mayring, 2015). The findings of the study are reported based on the tasks related to class level and learning domains. This involves determining the cognitive demand levels associated with each task. In the classification by Smith and Stein

(1998), each level is coded as follows: a) Memorization (Low Cognitive Demand-1 [LCD1]), b) procedures without connections (Low Cognitive Demand-2 [LCD2]), c) Procedures with connections (High Cognitive Demand-1 [HCD1]), and d) Doing mathematics (High Cognitive Demand-2 [HCD2]).

Each task was coded by both of researchers with specific codes whose examples are provided below. When researchers encountered disagreements in coding any task, they discussed their perspectives and decided on the most appropriate code through deliberation. To enhance the reliability of the data, 20 tasks, including those with disagreements, were reviewed by an expert familiar with the classification used in the study. The researchers then engaged in discussions with the expert about the coding decisions. After these discussions agreement-correlation coefficients was found as 92% between researchers, and 86% among researchers and expert (Miles & Huberman, 1994). In the following subsections, an overview of the tasks was briefly discussed, with specific task examples provided under the corresponding subheadings below.

Example Task for Memorization (LCD1)

Students are expected to recognize and repeat previously encountered and acquired information. In this algebraic task (Ciritci et al., 2019), students are asked to recall the definition: "Given that a, b, and n are natural numbers, in the expression an=b, a is called the base, and n, which indicates how many times a is multiplied, is called the exponent or power (Figure 2)." Students must then identify the base and exponent in given exponential numbers. This task does not involve relating the learned or repeated definitions, formulas, rules, or facts to their underlying meanings.

Figure 2. *Example Task for Memorization*

Identify only the bases and exponents of the given exponential numbers.

a) 83 b) 3¹⁰ c) 20¹⁰ d) 1999²⁰⁰⁰

Example Task for Procedures without Connections (LCD2)

In this algebraic task (Böge and Akıllı, 2019) students are expected to apply standardized and predictable formulas or algorithms to reach the solution (Figure 3). Specifically, they are required to find the solution of the equation and verify its accuracy. Solving the equation necessitates the use of algorithmic procedures. Students can solve previous experiences, explanations, and the sequence of tasks. However, there is no connection to the underlying concepts in this task. While explanations are provided during the solution process, these explanations pertain solely to the use of procedures.

Figure 3. *Example Task for Procedures without Connections*

Solve the given equations and verify the accuracy of your solutions.

a)
$$2a+5=13$$
 b) $3k-2=+7$ c) $2m-3=19$ d) $\frac{1}{2}y-6=4$ e) $2t+8=14$ f) $11=4x+3$ g) $3a+8=32$

Example Task for Procedures with connections (HCD1)

In this task (Böge and Akıllı, 2019) students are expected to draw conclusions about knowledge or methods, apply learned methods to new situations, choose appropriate methods for solving new problems, and transform or explain knowledge or methods (Figure 4). This algebraic task involves using a balance model to represent equations, establishing a connection between the model and mathematical concepts. Through this task, students perceive that the underlying concept of the balance model is the principle of equality.

Figure 4. *Example Task for Procedures with Connections*

Marbles on the Scale

- Let's place 6 marbles on the left pan of the scale and 10 marbles on the right pan.
- ❖ To balance the scale, let's add an object to the left pan. How can you write the algebraic expression for the balanced position of the scale, considering the object you placed as the unknown? Explain

Example Task for Doing Mathematics (HCD2)

In this task (Çağlayan et al., 2019) students are expected to independently identify a situation that is not explicitly stated in the task (Figure 5). Specifically, they are asked to create a pattern of shapes based on a numerical pattern, requiring them to engage in modelling. By following the steps provided in the task, students establish a relationship within the numerical pattern, derive a general rule, and explain their findings, and then effectively proving their conclusions. This task necessitates high level of cognitive demand as it requires students to make connections, generalize their findings, abstract, symbolize, and ultimately prove their results.

Figure 5.

Example Task for Doing Mathematics

Based on the pattern: 1x4, 2x4, 3x4,

- a) Find the values of the first 4 steps.
- b) Find the value of the 80th step and explain how you arrived at your answer.
- c) Create a shape pattern that corresponds to the given number pattern

Ethical Permits of Research:

In this study, all the rules specified to be followed within the scope of "Higher Education Institutions Scientific Research and Publication Ethics Directive" were complied with. None of the actions specified under the heading "Actions Contrary to Scientific Research and Publication Ethics", which is the second part of the directive, have been taken.

Ethics Committee Permission Information:

Ethics committee approval was not necessary for this research, as it does not involve human or animal subjects, and the data sources are publicly accessible textbooks. The journal's writing guidelines, publication principles, research and publication ethics rules, and journal ethics regulations were adhered to in this article. Any responsibility for potential violations related to the article lies with the author.

Findings

According to the findings of the study, examples of mathematical tasks found in the textbooks correspond to each level of the mathematical task classification proposed by Smith and Stein (1998). The details of these mathematical tasks and their distributions by grade level were presented in the following section. Additionally, their distributions according to learning areas were also discussed.

Findings for 5th Grade Textbook

The findings from the analysis of the 5th grade mathematics textbook tasks reveal a distribution across different cognitive demand levels, categorized into Low Cognitive Demand [LCD] and High Cognitive Demand [HCD] (see Table 1).

Table 1.Statistics for Cognitive Demand Levels of Tasks in 5th Grade Textbook

	Low co	ognitive	demand		High cognitive demand					
Learning area	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
Numbers and operations Algebra	19	9.69	84	42.86	68	34.69	25	12.76	196	52.69
Geometry and measurement	42	26.25	56	35.0 0	51	31.88	11	6.88	160	43.01
Data analysis Probability	3	18.75	6 -	37.50	6 -	37.50	1 -	6.25	16	4.30
Total	64	17.20	146	39.25	125	33.60	37	9.95	372	100

Within the Numbers and Operations learning area, the majority of tasks fall under the LCD2 category (42.86%), with a significant portion also classified as HCD1 (34.69%). In the geometry and measurement area, tasks are predominantly in the LCD2 (35.00%) and HCD1 (31.88%) categories. For data analysis, tasks are evenly distributed between LCD2 (37.50%) and HCD1 (37.50%). Overall, across all learning areas, 56.45% of the tasks are classified as low cognitive demand (LCD1 and LCD2), while 43.55% are high cognitive demand (HCD1 and HCD2). This indicates that while there is a greater prevalence of lower cognitive demand tasks, a substantial proportion of tasks also require higher cognitive engagement.

Findings for 6th Grade Textbook

The analysis of cognitive demand levels in the 6th grade mathematics textbook (see Table 2) reveals that a significant portion of tasks falls under low cognitive demand categories (LCD1 and LCD2), accounting for 28.90% and 19.46% of the tasks, respectively. In the numbers and operations learning area, tasks are almost equally distributed between LCD1 (33.46%) and HCD1 (33.46%), with a notable presence of LCD2 (20.46%) and HCD2 (12.64%). In the Algebra learning area, the majority of tasks are categorized as LCD1 (62.07%), followed by HCD1 (20.69%), LCD2 (13.79%), and HCD2 (3.45%). Geometry and measurement tasks show a more balanced distribution with 17.32% in LCD1, 18.99% in LCD2, 34.64% in HCD1, and 29.05% in HCD2. For data analysis, tasks are distributed with 26.19% in LCD1, 19.05% in LCD2, 35.71% in HCD1, and 19.05% in HCD2. There are no tasks recorded under Probability. Overall, 33.33% of the tasks are classified as HCD1, indicating a substantial effort to incorporate high cognitive demand tasks, while 18.30% are categorized as HCD2.

Table 2.Statistics for Cognitive Demand Levels of Tasks in 6th Grade Textbook

	L	ow cogni	tive dem	and	Hi	gh cogn				
Learning area	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
Numbers and operations	90	33.46	55	20.46	90	33.46	34	12.64	269	51.83
Algebra	18	62.07	4	13.79	6	20.69	1	3.45	29	5.59
Geometry and measurement	31	17.32	34	18.99	62	34.64	52	29.05	179	34.49
Data analysis	11	26.19	8	19.05	15	35.71	8	19.05	42	8.09
Probability		-		-	-	-		-		-
Total	150	28.90	101	19.46	173	33.33	95	18.30	519	100

Findings for 7th Grade Textbook

The analysis of cognitive demand levels in the 7th grade mathematics textbook (see Table 3) reveals that a significant portion of tasks fall under low cognitive demand categories, with LCD1 comprising 9.22% and LCD2 comprising 50.31% of the tasks. In the numbers and operations learning area, the majority of tasks are classified as LCD2 (55.53%), followed by HCD1 (26.13%), LCD1 (9.55%), and HCD2 (8.79%), making up 83.44% of the total tasks in this category.

For algebra, tasks are distributed among LCD1 (11.90%), LCD2 (21.43%), HCD1 (28.57%), and HCD2 (38.10%), constituting 8.81% of the total tasks. Geometry and measurement tasks show a varied distribution with no tasks in LCD1, but 25.81% in LCD2, 58.06% in HCD1, and 16.13% in HCD2, representing 6.50% of the total tasks. Data Analysis tasks are limited, with 16.67% in LCD1, 33.33% in LCD2, and 50% in HCD1, making up 1.26% of the total tasks. Overall, 59.53% of the tasks are classified as low cognitive demand and 40.47% as high cognitive demand.

Table 3.Statistics for Cognitive Demand Levels of Tasks in 7th Grade Textbook

	Lo	w cognit	ive dema	ınd	Hig	gh cognit				
Learning area	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
Numbers and operations	38	9.55	221	55.53	104	26.13	35	8.79	398	83.44
Algebra	5	11.90	9	21.43	12	28.57	16	38.10	42	8.81
Geometry and measurement	0	0.00	8	25.81	18	58.06	5	16.13	31	6.50
Data analysis	1	16.67	2	33.33	3	50	0	0.00	6	1.26
Probability		-		-	-	-	-	-		_
Total	44	9.22	240	50.31	137	28.72	56	11.74	477	100

Findings for 8th Grade Textbook

The distribution of cognitive demand levels within the tasks in the 8th mathematics textbook demonstrates a varied spread across different categories (see table 4).

Table 4.Statistics for Cognitive Demand Levels of Tasks in 8th Grade Textbook

	Lo	w Cognit	ive Dem	and	Hig	gh Cogni				
Learning area	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
Numbers and operations	3	5.08	28	47.46	8	13.56	20	33.90	59	13.59
Algebra	16	8.47	86	45.50	63	33.33	24	12.70	189	43.55
Geometry and measurement	0	0.00	71	50.71	49	35.00	20	14.29	140	32.26
Data analysis	3	15.79	3	15.79	4	21.05	9	47.37	19	4.38
Probability	5	18.53	6	22.22	9	33.33	7	25.93	27	6.22
Total	27	6.22	194	44.70	133	30.65	80	18.43	434	100

In the numbers and operations section, tasks primarily fall into the LCD2 category, making up 47.46% of the tasks, followed by HCD2 at 33.90%. For Algebra, the majority of tasks are categorized as LCD2 (45.50%), with a substantial proportion also classified as HCD1 (33.33%). In geometry and measurement, the highest percentage of tasks is in the LCD2 category (50.71%), while HCD1 and HCD2 tasks account for 35.00% and 14.29%, respectively. Data Analysis tasks are evenly distributed between LCD1, LCD2, HCD1, and but not HCD2, each representing 15.79%, 15.79%, 21.05%, and 47.37% respectively. In the Probability section, tasks are not predominantly low cognitive demand, with LCD1 and LCD2 constituting 18.53% and 22.22% of the tasks, while HCD1 and HCD2 make up 33.33% and 25.93%. In general, the total distribution reveals that 44.70% of tasks are classified under LCD2, 30.65% under HCD1, 18.43% under HCD2, and %6.22 under LCD1.

Overall Findings by Grade Level

The distribution of cognitive demand levels across different grade levels (see Table 5) reveals distinct patterns. In the 5th grade, tasks are predominantly classified under LCD2 (39.25%), followed by HCD1 (33.60%), indicating a balanced mix of low and high cognitive demand tasks. The 6th grade shows a significant portion of tasks in the LCD1 category (28.90%), with HCD1 (33.33%) and HCD2 (18.30%) also well represented, suggesting a shift towards higher cognitive demands. For the 7th grade, the majority of tasks fall under LCD2 (50.31%), highlighting a strong focus on procedural tasks without connections, with notable proportions in HCD1 (28.72%) and HCD2 (11.74%). In the 8th grade, tasks are similarly

distributed with a large percentage in LCD2 (44.70%) and significant representation in HCD1 (30.65%) and HCD2 (18.43%), reflecting a continued emphasis on higher cognitive demand tasks. Overall, the total distribution indicates that 37.79% of tasks are classified under LCD2, 31.52% under HCD1, and 14.87% under HCD2.

Table 5.Statistics for Overall Cognitive Demand Levels by Grade Level

	Lo	w cognit	ive dema	and	Hi	gh cognit				
Grade level	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
5 th grade	64	17.20	146	39.25	125	33.60	37	9.95	372	20.64
6 th grade	150	28.90	101	19.46	173	33.33	95	18.30	519	28.80
$7^{ m th}$ grade	44	9.22	240	50.31	137	28.72	56	11.74	477	26.47
8 th grade	27	6.22	194	44.70	133	30.65	80	18.43	434	24.08
Total	285	15.82	681	37.79	568	31.52	268	14.87	1802	100

Overall Findings by Learning Area

The analysis of cognitive demand levels across different learning areas (see Table 6) reveals notable variations. In the numbers and operations category, a significant portion of tasks fall under LCD2 (42.08%), followed by HCD1 (29.28%) and LCD1 (16.27%), with HCD2 tasks constituting 12.36% of the total. This area comprises 51.17% of all tasks. In the Algebra category, LCD2 tasks dominate with 38.08%, while HCD1 and HCD2 tasks account for 31.15% and 15.77% respectively, and LCD1 tasks make up 15.00%. Algebra tasks represent 14.43% of the total tasks analysed.

Table 6.Statistics for Overall Cognitive Demand Levels by Learning Area

	Lo	w cognit	ive dema	and	Hiş	gh cognit				
Learning area	LCD1	%	LCD2	%	HCD1	%	HCD2	%	Total	%
Numbers and operations	150	16.27	388	42.08	270	29.28	114	12.36	922	51.17
Algebra	39	15.00	99	38.08	81	31.15	41	15.77	260	14.43
Geometry and measurement	73	14.31	169	33.14	180	35.29	88	17.25	510	28.30
Data analysis	18	21.69	19	22.89	28	33.73	18	21.69	83	4.61
Probability	5	18.53	6	22.22	9	33.33	7	25.93	27	1.50
Total	285	15.82	681	37.79	568	31.52	268	14.87	1802	100

For geometry and measurement, the distribution shows a higher percentage of HCD1 tasks (35.29%), followed by LCD2 (33.14%), HCD2 (17.25%), and LCD1 (14.31%) (Table 6). This category makes up 28.30% of the total tasks. In the Data Analysis category, the majority of tasks are in the HCD1 (33.73%) and HCD2 (21.69%) categories, with LCD1 and LCD2 each representing 21.69% and 22.89% respectively. Data Analysis tasks account for 4.61% of the total. In the Probability category, tasks are evenly distributed among LCD1 (18.53%), LCD2 (22.22%), HCD1 (33.33%), and HCD2 (25.93%), although this area constitutes only 1.50% of the total tasks (Table 6). Overall, the total distribution indicates that 37.79% of tasks are categorized under LCD2, 31.52% under HCD1, and 14.87% under HCD2.

As a summary, the current study found that mathematical tasks in textbooks correspond to the classification levels proposed by Smith and Stein (1998). For the 5th grade, tasks are primarily of LCD, though a significant portion requires HCD. The 6th grade textbook shows a notable presence of high cognitive demand tasks, particularly in the Algebra learning

area. In the 7th grade, there is a strong focus on procedural tasks with lower cognitive demands, yet a considerable number of tasks also necessitate higher cognitive skills. The 8th grade textbook maintains a balanced distribution of tasks across different cognitive demand levels. Overall, the findings indicate a mix of low and high cognitive demand tasks across all grades, with variations depending on the learning area, such as Numbers and Operations or Geometry and Measurement.

Discussion and Conclusion

To briefly summarize the findings of the study, the tasks with the highest proportions are as follows: procedures without connections tasks (LCD2) – 681 tasks (37.79%), followed by Procedures with Connections tasks (HCD1) - 568 tasks (31.52%). The task with the lowest proportion is doing mathematics tasks (HCD2) – 268 tasks (14.87%). High cognitive demand tasks are the most frequently observed in the 6th grade textbook, with Procedures with Connections (HCD1) accounting for 173 tasks (33.33%) and doing mathematics (HCD2) for 95 tasks (18.30%), making a total of 268 tasks (51.63%). However, the 6th grade textbook also has the highest proportion of memorization tasks (LCD1), 150 tasks (28.90%). The grade level with the lowest cognitive demand tasks is the 7th grade textbook, containing 44 memorization tasks (LCD1) (9.22%) and 240 procedures without connections tasks (LCD2) (50.31%), making a total of 284 tasks (59.53%). The proportion of tasks with high cognitive demand is highest in the probability learning area; however, due to the very low number of objectives and tasks, this might lead to incorrect interpretations (16 tasks, 59.26%). High cognitive demand tasks are notably present in the data analysis and geometry and measurement learning areas (46 tasks, 55.42% and 268 tasks, 52.54%, respectively). Conversely, the learning area with the highest proportion of low cognitive demand tasks is numbers and operations (538 tasks, 58.35%). Another interesting point is that, in the 7th and 8th grade textbooks, none of the tasks related to geometry and scale were found to be at the memorization level. This may be due to the focus on procedures and relationships at these grade levels. Additionally, information that requires direct memorization, such as the number of sides, vertices, or the sum of interior angles in geometric shapes, is typically covered in earlier grades, even at the primary level. Similarly, the distribution of tasks in the 7th grade textbook on data analysis might have been difficult to adjust due to the presence of only one learning outcome.

These findings underscore the need for a balanced approach in the design of mathematics textbooks. While procedural fluency is important, the integration of tasks that promote higher-order thinking and problem-solving skills is essential for comprehensive mathematical education. This balance can better prepare students for real-world applications of mathematics and foster a deeper understanding of the subject (Stein & Smith, 1998). Moreover, the findings of this study provide significant insights into the cognitive demand levels of tasks in mathematics textbooks. The overemphasis on procedural tasks, both with and without connections, suggests a missed opportunity to engage students in more complex and meaningful mathematical thinking (Bozkurt & Yılmaz, 2020). The alignment with Bozkurt and Yılmaz's (2020) results state that the tasks in these textbooks are largely concentrated at the levels of procedures without connections (LCD2) and Procedures with Connections (HCD1). This indicates a predominance of tasks that focus on procedural skills, both with and without

conceptual connections. This trend highlights an ongoing challenge in mathematics education: the need to ensure that students are not only proficient in basic procedures but also capable of engaging in complex problem-solving and critical thinking (Henningsen & Stein, 1997).

The findings of present study are consistent with those of Ubuz and Sarpkaya (2014) as well as Engin and Sezer (2016), both of which highlighted that the proportion of doing mathematics tasks (HCD2) in textbooks is relatively low. This underrepresentation of highlevel cognitive demand tasks suggests a potential area for improvement in textbook content to better support the development of students' higher-order thinking skills. Incorporating more tasks that require students to make connections and engage with mathematical concepts deeply can enhance their understanding and retention of the material (Kilpatrick et al., 2001).

Furthermore, the findings of present study align with those of Reçber and Sezer (2018), who found that the proportion of high cognitive demand tasks in 8th grade mathematics textbooks was lower than expected in the middle school mathematics curriculum. Similarly, the results of the current study indicate a limited presence of tasks that require higher-order thinking skills, further emphasizing the need for greater inclusion of cognitively demanding tasks in mathematics education (Stein & Smith, 1998). Additionally, the findings of Polat and Dede (2023) revealed that the tasks in the algebra learning domain of the analysed mathematics textbooks from the 2000s to today were generally of low cognitive demand. This indicates that these tasks do not require students to engage in high-level cognitive thinking and problem-solving skills. These results align with the findings of the current study, suggesting that both in the past and present, the cognitive demand levels of the tasks remain low.

The expectation for the use of high cognitive demand tasks in the middle school mathematics curriculum is not reflected in the content of the textbooks. Despite the curriculum emphasizing the importance of engaging students with tasks that require higher-order thinking and deep understanding, the actual materials provided in the textbooks tend to focus more on lower cognitive demand tasks. This discrepancy indicates a gap between the educational goals set by the curriculum and the resources available to teachers and students. This gap between curricular goals and textbook content suggests that educators may need to supplement textbooks with additional resources and tasks that promote higher cognitive engagement (Schmidt et al., 2002). Research has shown that tasks with high cognitive demand are essential for developing students' mathematical reasoning, problem-solving, and critical thinking skills. These skills are crucial for success in advanced mathematics and Science, Technology, Engineering and Math [STEM]-related fields, as well as for everyday problemsolving and decision-making (NRC, 2012). However, the lack of such tasks in the textbooks suggests that students may not be adequately challenged to reach their full potential in mathematics. Ensuring that textbooks include a greater proportion of high cognitive demand tasks can help bridge this gap and provide students with a more well-rounded mathematics education. According to a study by Ni et al. (2018), the frequency of mathematical tasks involving high cognitive demand did not predict cognitive learning outcomes but positively influenced students' interest in learning mathematics and classroom participation, highlighting the need for such tasks in educational materials.

In conclusion, the study highlights the need for a more strategic approach in the design and selection of tasks in mathematics textbooks. By increasing the proportion of high cognitive demand tasks, textbooks might better support the development of students' higher-order thinking skills and prepare them for future academic and professional challenges. This shift is critical for aligning educational resources with curricular goals and enhancing the overall quality of mathematics education.

Recommendations

The findings and discussions of this study emphasize the need for a more balanced inclusion of tasks that promote deep cognitive engagement and conceptual understanding. Future textbook revisions should consider increasing the proportion of high cognitive demand tasks to foster a more comprehensive mathematical education that not only builds procedural proficiency but also enhances students' ability to engage in complex problem-solving and critical thinking.

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Contribution Rate of Researchers

Author 1: 50%

Author 2: 50%

Conflict Statement

The authors declare that there is no conflict of interest in the research.

Ortaokul Matematik Ders Kitaplarındaki Görevlerin Bilişsel İstem Düzeylerine Göre Analizi



Özet

Bu çalışma, ortaokul matematik ders kitaplarındaki matematiksel görevleri, bilişsel istem düzeylerine göre ayrıntılı bir biçimde incelemeyi ve sınıflandırmayı amaçlamaktadır. Bu görevlerin, öğrencilerin eleştirel düşünme, problem çözme ve matematiksel akıl yürütme becerilerini geliştirmeye yönelik olarak tasarlanan eğitim hedefleriyle nasıl bir uyum gösterdiğini ortaya çıkarmayı hedeflemektedir. Çalışmanın deseni doküman analizidir ve ders kitaplarında yer alan matematiksel görevlerin niteliğini derinlemesine incelemektedir. Bu çalışma kapsamında, 5., 6., 7. ve 8. sınıf ders kitaplarında yer alan matematiksel görevler, Stein ve Smith (1998) tarafından geliştirilen matematiksel görev sınıflandırma çerçevesi esas alınarak değerlendirilmiştir. Veriler, içerik analizi yöntemiyle incelenmiş ve görevlerin bilişsel düzeylerine göre dağılımları değerlendirilmiştir. Sonuçlar, ders kitaplarında yer alan matematiksel görevlerin çoğunlukla bağlantısız yöntemler ve bağlantılı yöntemler düzeylerinde yoğunlaştığını göstermiştir. Buna karşın, özellikle üst bilişsel beceriler gerektiren matematik yapma görevlerinin oranının düşük olduğu belirlenmiştir. Bu nedenle, gelecekteki ders kitabı revizyonlarının, öğrencilerin sadece işlemsel becerilerini değil, aynı zamanda karmaşık problem çözme ve eleştirel düşünme yeteneklerini de geliştirecek daha fazla bilişsel istem gerektiren görevleri içerecek şekilde düzenlenebilir.

Anahtar Kelimeler: Bilişsel istem, matematiksel görev, ders kitabı, ortaokul matematiği, etkinlik.

Giriş

1980'lerin ortalarından itibaren matematik eğitiminde etkisini artıran yapılandırmacı vaklasım, temel olarak öğrenci merkezliliği ve öğrencinin aktif katılımını vurgulamaktadır. Bu yaklaşıma göre, öğrencilerin, mevcut bilgileri üzerine yeni bilgileri inşa ederek öğrenmelerini kendilerinin yapılandırmaları beklenir (Toluk-Uçar, 2020; Van de Walle vd., 2023). Boston vd. (2017), yapılandırmacı yaklaşımın hedeflerine ulaşmada ve öğrencilerin anlamlı matematik öğrenmelerini desteklemede, problem çözme ve mantıksal akıl yürütmeyi tesvik eden etkinliklerin öğretim sürecine dahil edilmesinin kilit unsur olduğunu savunmaktadır. Etkinlikler, öğrencilerin aktif katılımını tesvik eden sürecler olarak tanımlanabileceği gibi, öğretmen tarafından seçilen ve öğrenci tarafından gönüllü olarak gerçekleştirilen bireysel veya bir dizi görev olarak da tanımlanabilir (Dede vd., 2020). Matematik etkinlikleri, öğrencilerin matematik derslerindeki öğrenme ve deneyimlerini şekillendirir (Johnson vd., 2017). İyi organize edilmiş ve etkili bir şekilde uygulanan etkinlikler, öğrencilerin sınıf içinde bilgi, beceri ve anlayışlarını yansıtmalarına olanak tanır (Smith & Stein, 2011). Kilpatrick vd. (2001), net ve motive edici etkinliklerin, öğrencilerin mantıksal akıl yürütme becerilerini gösterebilmeleri için gerekli üç koşuldan biri olduğunu belirtmektedir (diğer ikisi yeterli bilgiye sahip olmak ve içeriğe aşina olmaktır). Matematik eğitimi literatüründe "etkinlik" terimi farklı biçimlerde karşımıza çıkmaktadır.

"Alıştırma", "Görev" ve "Etkinlik" terimleri genellikle birbirinin yerine kullanılsa da tam anlamıyla aynı şeyi ifade etmemektedir. Alıştırmalar, öğrencilere genellikle yazılı halde sunulur ve öğretmen kontrolü çok yüksektir. Ayrıca, işlemsel becerilere odaklanır ve tekrara dayalı öğrenmeyi destekler. (Cichy vd., 2020; Foster, 2018; Watson & Mason, 2006). Görevler ise düşük öğretmen kontrolü olması hem işlemsel hem kavramsal bilgiye odaklanma ve sosyal süreçlerle öğrenmenin tekrarı gibi özelliklerle tanımlanır (Chapman, 2013; Foster, 2013). Etkinlikler, öğrencinin kontrolünde olup öğretmenin rehber rolünde olduğu, aktif katılım gerektiren, keşfetmeyi teşvik eden ve önceki öğrenmeler üzerine yeni konular inşa etmeye odaklanan süreçlerdir (Antonijević, 2016; Ponte vd., 2014). Bu üç terimin kullanımı ve işlevi göz önünde bulundurulduğunda, etkinlikler, öğrencilerin belirli pedagojik yaklaşımlar ve matematiksel görevler eşliğinde belirli matematiksel fikirlere odaklanmalarını sağlayan öğrenme etkinlikleri olarak tanımlanabilir. Bu bağlamda, "etkinlik" teriminin temel dayanağı, matematiksel öğrenme ortamlarında ve öğretim süreçlerinde, öğrencilerin matematiksel düşünme, akıl yürütme, modelleme ve anlama becerilerini geliştirmeye yardımcı olacak fırsatların ve koşulların sağlanmasıdır (Seah & Horne, 2021; Stein vd., 1996; Wess vd., 2021).

Bilişsel düzeylerine veya öğrencilerden beklenen bilişsel istemlerine göre görevleri tanımlamak ve sınıflandırmak, etkinliklerin oluşturulması ve değerlendirilmesi sürecinde faydalı yöntemlerden biridir. Burada bahsedilen bilişsel düzey, bir öğrencinin belirli bir etkinliği tamamlaması için gereken bilişsel süreçlere atıfta bulunmaktadır. Matematiksel görevler, rutin alıştırmalardan, karmaşık ve zorlu problemlere kadar bir süreklilik göstermektedir. Doyle'un (1988) çalışmasına dayanarak, Stein ve meslektaşları, matematiksel görev türlerini ve bu görevleri çözmek için gereken düşünme düzeylerini sınıflandırmak amacıyla bir dizi çalışma yapmıştır (Stein vd., 1996; Stein & Smith, 1998). Smith ve Stein (1998), matematiksel görevleri iki düzeyde sınıflandırmıştır: düşük düzey ve yüksek düzey. Düşük düzey görevler, ezberleme ve bağlantısız yöntemler olarak ayrılmışken, yüksek düzey görevler, bağlantılı yöntemler ve matematik yapma olarak ayrılmıştır.

Bu çalışma, ortaokul matematik ders kitaplarındaki görevleri analiz etmeyi ve bilişsel istem düzeylerine göre sınıflandırmayı amaçlamaktadır. Bu yolla, ders kitaplarında sunulan görevlerin niteliği ve bu görevlerin eleştirel düşünme ve problem çözme becerilerini geliştirmeye yönelik eğitim hedefleriyle uyumu hakkında bilgi sunmak amaçlanmaktadır. Bu çalışmanın bulguları, ders kitaplarının bu temel becerilerin gelişimini ne ölçüde desteklediğini veya engellediğini anlamaya katkı sağlayacaktır.

Yöntem

Bu çalışmanın amacı, ortaokul matematik ders kitaplarındaki görevleri bilişsel istem düzeylerine göre analiz etmek ve sınıflandırmaktır. Çalışmanın deseni, incelenen konu hakkında detaylı bilgi sağlamak için yazılı veya basılı materyallerin incelenmesini içeren bir veri toplama süreci olan (Bowen, 2009) doküman analizidir. Bu çalışmada, 5., 6., 7. ve 8. sınıf matematik ders kitaplarındaki matematiksel görevler, Smith ve Stein (1998) tarafından geliştirilen sınıflandırma çerçevesine dayanarak değerlendirilmiştir. Veriler, nitel verilerden sonuç çıkarma tekniklerinden biri olan içerik analizi yöntemi kullanılarak incelenmiştir.

Smith ve Stein (1998) tarafından yapılan sınıflandırmada her seviye şu şekilde kodlanmıştır: a) Ezberleme (LCD1), b) Bağlantısız yöntemler (LCD2), c) Bağlantılı yöntemler (HCD1) ve d) Matematik yapma (HCD2). Ezberleme, bilgilerin basitçe hatırlanmasıdır.

Bağlantısız yöntemler, algoritmaların uygulanmasını gerektirir ama kavramsal bir anlayışa dayanmaz. Bağlantılı yöntemler, yapılan işlemleri kavramsal bir anlayışla ilişkilendirir. Matematik yapma, derinlemesine düşünme ve problem çözme yetenekleri gerektirir. Düzeyler ilerledikçe, daha fazla zihinsel çaba ve kavramsal anlayış devreye girer.

Her görev, iki araştırmacı tarafından belirli bir kodla kodlanmıştır. Kodlamada anlaşmazlık yaşandığında, araştırmacılar görüşlerini tartışarak en uygun koda karar vermişlerdir. Verilerin güvenilirliğini artırmak için, anlaşmazlık içerenler de dahil olmak üzere 20 görev, çalışmada kullanılan sınıflandırmaya aşina olan bir uzman tarafından gözden geçirilmiştir. İlgili uzman ve araştırmacılar daha sonra kodlama kararlarını tartışmışlardır. Araştırmacılar arasındaki uyum-korelasyon katsayısı %92, araştırmacılar ve uzman arasındaki ise %86'dır (Miles & Huberman, 1994).

Araştırmanın Etik İzinleri:

Bu çalışmada "Yükseköğretim Kurumları Bilimsel Araştırma ve Yayın Etiği Yönergesi" kapsamında uyulması gerektiği belirtilen tüm kurallara uyulmuştur. Yönergenin ikinci bölümü olan "Bilimsel Araştırma ve Yayın Etiğine Aykırı Eylemler" başlığı altında belirtilen eylemlerin hiçbiri gerçekleştirilmemiştir.

Etik Kurul İzin Bilgileri:

Bu araştırma, insan veya hayvan denekler içermediği ve veri kaynakları kamuya açık ders kitapları olduğu için etik kurul onayı gerektirmemektedir. Bu makalede, dergi yazım kuralları, yayın ilkeleri, araştırma ve yayın etiği kuralları ve dergi etik kuralları titizlikle takip edilmiştir. Makale ile ilgili olarak ortaya çıkabilecek herhangi bir ihlalin sorumluluğu yazarlara aittir.

Bulgular

5. sınıf matematik ders kitabındaki görevlerin analizi, Düşük Bilişsel İstem [LCD] ve Yüksek Bilişsel İstem [HCD] düzeylerinde dağılım göstermektedir. Sayılar ve İşlemler öğrenme alanında görevlerin çoğunluğu LCD2 (%42,86) düzeyinde yer alırken, önemli bir kısmı da HCD1 (%34,69) olarak sınıflandırılmıştır. Geometri ve ölçme alanında görevler ağırlıklı olarak LCD2 (%35.00) ve HCD1 (%31,88) düzeyindedir. Veri analizi alanında görevler LCD2 (%37,50) ve HCD1 (%37,50) düzeyinde eşit dağılım göstermektedir. Genel olarak, tüm öğrenme alanlarında görevlerin %56,45'i düşük bilişsel istem, %43,55'i ise yüksek bilişsel istem düzeyindedir. Bu durum, daha düşük bilişsel istemli görevlerin yaygın olmasına rağmen, önemli bir oranda yüksek bilişsel katılım gerektiren görevlerin de bulunduğunu göstermektedir.

6. sınıf matematik ders kitabındaki bilişsel istem düzeylerinin analizi, görevlerin önemli bir kısmının düşük bilişsel istem düzeyinde (LCD1 %28,90 ve LCD2 %19,46) yer aldığını göstermektedir. Sayılar ve işlemler alanında, görevler LCD1 (%33,46) ve HCD1 (%33,46) arasında eşit olarak dağılmıştır. Cebir alanında görevlerin çoğunluğu LCD1 (%62,07) olarak sınıflandırılmıştır. Geometri ve ölçme ile veri analizi alanlarında görevler daha dengeli bir dağılım göstermektedir. Genel olarak, görevlerin %33,33'ü HCD1, %18,30'u ise HCD2 olarak sınıflandırılmıştır.

7. sınıf matematik ders kitabındaki bilişsel istem düzeylerinin analizi, görevlerin önemli bir kısmının düşük bilişsel istem düzeylerinde olduğunu göstermektedir; LCD1 %9,22 ve LCD2 %50,31. Sayılar ve işlemler alanında görevlerin çoğunluğu LCD2 (%55,53), ardından HCD1 (%26,13) olarak sınıflandırılmıştır. Cebir alanında görevler LCD1 (%11,90), LCD2 (%21.43), HCD1 (%28,57) ve HCD2 (%38,10) olarak dağılmıştır. Geometri ve ölçme alanında görevler LCD1 içermemekle birlikte, LCD2 (%25,81), HCD1 (%58,06) ve HCD2 (%16,13) olarak dağılım göstermektedir. Veri analizi alanında görevler sınırlıdır; LCD1 (%16,67), LCD2 (%33,33) ve HCD1 (%50). Genel olarak, görevlerin %59,53'ü düşük bilişsel istem, %40,47'si yüksek bilişsel istem düzeyindedir.

8. sınıf matematik ders kitabındaki görevlerin bilişsel istem düzeylerinin dağılımı, farklı düzeyler arasında çeşitlilik göstermektedir. Sayılar ve işlemler bölümünde görevlerin çoğunluğu LCD2 düzeyinde (%47,46) yer alırken, bunu HCD2 düzeyindeki görevler (%33,90) izlemektedir. Cebir bölümünde görevlerin çoğu LCD2 (%45,50) ve HCD1 (%33,33) olarak sınıflandırılmıştır. Geometri ve Ölçme bölümünde görevlerin en yüksek yüzdesi LCD2 düzeyinde (%50,71), HCD1 ve HCD2 görevleri ise sırasıyla %35.00 ve %14,29'dur. Veri analizi görevleri düşük düzey beklentilerde LCD1 (%15,79) ve LCD2 (%15,79) olacak şekilde eşit dağıtılmıştır, yüksek düzey beklentiler içeren görevlerden HCD1 (%21,05) düşük düzey beklentiler içeren görevlerden HCD1 (%21,05) düşük düzey beklentiler içeren görevlere yakındır fakat matematik yapma düzeyindeki HCD2 (%47,37) diğerlerinden daha fazladır. Olasılık bölümünde görevler, genellikle düşük bilişsel istem düzeylerinde değildir; LCD1 %18,53 ve LCD2 %22.22, HCD1 %33,33 ve HCD2 %25,93 oranındadır. Genel olarak, görevlerin %44,70'i LCD2, %30,65'i HCD1 ve %18,43'ü HCD2 olarak sınıflandırılmıştır.

Farklı sınıf seviyelerindeki bilişsel istem düzeylerinin dağılımı belirgin bir örüntü göstermemektedir. 5. sınıfta görevler ağırlıklı olarak LCD2 (%39,25) ve HCD1 (%33,60) olarak sınıflandırılmıştır. 6. sınıfta, görevlerin büyük bir kısmı HCD1 (%33,33) düzeyinde olup, diğer görevlerin oranları ise sırasıyla LCD1 (%28,90), LCD2 (%19,46) ve HCD2(%18,30) şeklindedir. 7. sınıfta görevlerin çoğu LCD2 (%50,31) olup, HCD1 (%28,72) ve HCD2 (%11,74) oranları dikkat çekmektedir. 8. sınıfta, görevler benzer şekilde LCD2 (%44,70), HCD1 (%30,65) ve HCD2 (%18,43) olarak dağılım göstermektedir. Genel olarak, görevlerin %37,79'u LCD2, %31,52'si HCD1 ve %14,87'si HCD2 olarak sınıflandırılmıştır,

Farklı öğrenme alanlarındaki bilişsel istem düzeylerinin analizi önemli farklılıklar göstermektedir. Sayılar ve işlemler alt öğrenme alanında, görevlerin büyük bir kısmı LCD2 (%42.08), ardından HCD1 (%29,28) ve LCD1 (%16,27) olarak dağılım göstermektedir, HCD2 görevleri toplamın %12,36'sını oluşturmaktadır. Bu alan, tüm görevlerin %51,17'sini kapsamaktadır. Cebir alt öğrenme alanında, LCD2 görevleri %38,08 ile baskın olup, HCD1 ve HCD2 görevleri sırasıyla %31,15 ve %15,77, LCD1 görevleri ise %15,00'dir. Cebir görevleri toplam görevlerin %14,43'ünü temsil etmektedir. Geometri ve ölçme alt öğrenme alanında, HCD1 düzeyindeki görevler (%35,29) daha yüksek bir yüzde gösterirken, LCD2 (%33,14), HCD2 (%17,25) ve LCD1 (%14,31) düzeyindeki görevler sırasıyla yer almaktadır. Bu alt öğrenme alanı toplam görevlerin %28,30'unu içermektedir. Veri analizi alt öğrenme alanında, görevlerin çoğunluğu HCD1 (%33,73) ve HCD2 (%21,69) düzeylerinde olup, LCD1 ve LCD2 görevleri sırasıyla %21,69 ve %22,89'dur. Veri analizi görevleri toplamın %4,61'ini

kapsamaktadır. Olasılık alt öğrenme alanında, görevler LCD1 (%18,53), LCD2 (%22,22), HCD1 (%33,33) ve HCD2 (%25,93) yüzde olarak çok farklı dağılmış gibi görünse de bu alan toplam görevlerin yalnızca %1.50'sini oluşturmaktadır her bir düzey için ders kitaplarındaki görev sayıları birbirine yakındır. Genel olarak, görevlerin %37,79'u LCD2, %31,52'si HCD1 ve %14,87'si HCD2 olarak sınıflandırılmıştır.

Tartışma ve Sonuç

Bu çalışmanın bulguları, 5., 6., 7. ve 8. sınıf matematik ders kitaplarındaki görevlerin bilişsel istem düzeylerine dair önemli tespitler sunmaktadır. Bozkurt ve Yılmaz (2020) ile uyumlu olarak, bu kitaplarda görevlerin büyük ölçüde bağlantısız yöntemler (LCD2) – 681 görev (37.79%) ve bağlantılı yöntemler (HCD1) – 568 görev (31.52%) seviyelerinde yoğunlaştığını göstermektedir. Bu durum hem kavramsal bağlantılarla hem de bağlantısız işlemsel becerilere odaklanan görevlerin baskın olduğunu göstermektedir. Matematiksel görevlerin büyük bir çoğunluğunun, işlemsel becerilere dayalı görevlerden oluştuğu ve üst bilişsel düzeydeki görevlerin nispeten yetersiz kaldığı görülmektedir. Bu bulgular, matematik ders kitaplarının tasarımında dengeli bir yaklaşımın gerekliliğini vurgulamaktadır. İşlemsel akıcılık önemli olsa da üst düzey düşünme ve problem çözme becerilerini teşvik eden görevlerin entegrasyonu, kapsamlı bir matematik eğitimi için esastır. Bu denge, öğrencileri matematiğin gerçek dünya uygulamalarına daha iyi hazırlayabilir ve konuya daha derin bir anlayış kazandırabilir (Stein & Smith, 1998).

Ubuz ve Sarpkaya (2014) ile Engin ve Sezer (2016) çalışmalarında, ders kitaplarında üst düzey bilişsel becerileri gerektiren matematik yapma (HCD2) görevlerinin yeterli oranda yer almadığı tespit edilmiştir. Bu çalışmanın bulguları da benzer şekilde, ders kitaplarındaki matematik yapma görevlerinin oranının nispeten düşük olduğunu ortaya koymakta ve bu durumun öğrencilerin matematiksel düşünme becerilerini geliştirme açısından önemli bir eksiklik yarattığını vurgulamaktadır. Bu yüksek düzey bilişsel istem gerektiren görevlerin yeterince temsil edilmemesi, ders kitabı içeriğinde öğrencilerin üst düzey düşünme becerilerini geliştirmeye yönelik potansiyel bir iyileştirme alanı olarak görülebilir.

Ayrıca, bu çalışmanın bulguları, Reçber ve Sezer (2018) tarafından yapılan çalışmanın bulgularıyla da örtüşmektedir; yazarla çalışmalarında, 8. sınıf matematik ders kitaplarında yüksek bilişsel istem gerektiren görevlerin, ortaokul matematik müfredatındaki beklentilerin altında bir oranda bulunduğunu ortaya koymuştur. Benzer şekilde, bu çalışmanın sonuçları da üst düzey düşünme becerileri gerektiren görevlerin sınırlı bir şekilde yer aldığını göstererek, matematik eğitiminde bilişsel olarak zorlayıcı görevlerin daha fazla dahil edilmesi gerekliliğini vurgulamaktadır (Stein & Smith, 1998). Ek olarak, Polat ve Dede (2023) tarafından yapılan çalışmanın bulguları, 2000'li yıllardan bugüne kadar analiz edilen matematik ders kitaplarındaki cebir öğrenme alanında yer alan görevlerin genellikle düşük bilişsel istem düzeyinde olduğunu ortaya koymuştur. Bu durum, bu görevlerin öğrencilerin yüksek düzeyde bilişsel düşünme ve problem çözme becerilerini kullanmalarını gerektirmediğini göstermektedir. Bu sonuçlar, mevcut çalışmanın bulgularıyla örtüşmekte olup geçmişte ve günümüzde görevlerin bilişsel istem düzeylerinin düşük kaldığını işaret etmektedir.

Diğer taraftan, ortaokul matematik müfredatında yüksek bilişsel talep gerektiren görevlerin kullanımına yönelik beklenti, ders kitaplarının içeriğinde tam olarak karşılanmamaktadır. Müfredat, öğrencilerin üst düzey düşünme ve derin kavrayış gerektiren görevlerle meşgul olmalarının önemini vurgulasa da ders kitaplarında sunulan materyaller çoğunlukla düşük bilişsel talep gerektiren görevlere odaklanmaktadır. Bu tutarsızlık, müfredat tarafından belirlenen eğitim hedefleri ile öğretmenler ve öğrenciler için sunulan kaynaklar arasında bir boşluğun olduğunu göstermektedir. Ders kitaplarında bu tür görevlerin eksikliği, öğrencilerin matematikte tam potansiyellerine ulaşmak için yeterince zorlanmadıklarını göstermektedir. Ders kitaplarında yüksek bilişsel talep gerektiren görevlerin oranını artırmak, bu boşluğu kapatmaya yardımcı olabilir ve öğrencilere daha dengeli bir matematik eğitimi sunabilir.

Sonuç olarak, bu çalışma, matematik ders kitaplarında görevlerin tasarımında ve seçiminde daha stratejik bir yaklaşımın gerekliliğini vurgulamaktadır. Yüksek bilişsel talep gerektiren görevlerin oranını artırarak, ders kitapları öğrencilerin üst düzey düşünme becerilerini geliştirmelerine ve gelecekteki akademik ve mesleki zorluklara daha iyi hazırlanmalarına destek olabilir. Bu değişim, eğitim kaynaklarını müfredat hedefleriyle uyumlu hale getirmek ve matematik eğitiminin genel kalitesini artırmak için kritik bir öneme sahiptir.

Öneriler

Bu çalışmanın bulguları ve tartışmaları, derin bilişsel katılımı ve kavramsal anlayışı teşvik eden görevlerin daha dengeli bir şekilde dahil edilmesi gerektiğini vurgulamaktadır. Gelecekteki ders kitabı revizyonlarında, sadece işlem yapabilme yeterliliğini geliştirmekle kalmayıp, aynı zamanda öğrencilerin karmaşık problem çözme ve eleştirel düşünme becerilerini de artıran yüksek bilişsel istem gerektiren görevlerin oranının artırılması önerilmektedir.